WMML
Meet #4
Feb. 1, 2022

Name ____________________

School ____________________

Arithmetic and Number Theory

1) If 4 morbs are worth 3 meebs and 2 meebs are worth 5 marps, how many marps are worth the same as 10 morbs?

1. 18.75 marps

$$10 \text{ morbs} \cdot \frac{3 \text{ meebs}}{4 \text{ morbs}} = 7.5 \text{ meebs}$$

$$7.5 \text{ meebs} \cdot \frac{5 \text{ marps}}{2 \text{ meebs}} = 18.75 \text{ marps}$$

2) The price of a car is originally \$10,000. If the price decreased by 25%, then increased by 25%, and finally decreased by 25% again. What is the final price of the car?

2. \$7,031.25

$$\$10000(0.75) = \$7500$$

$$\$7500(1.25) = \$9375$$

$$\$9375(0.75) = \$7031.25$$

3) Find the units digit of $7^{42} + 42^{7}$.

3. 7

7^1 ends in 1, 7^2 ends in 9, 7^3 ends in 3, 7^4 ends in 1, then it repeats. So 7^{42} ends in 9.

42^7 has the same units digit as $2^7 = 128$, so it ends in an 8.

$9 + 8 = 17$, so $7^{42} + 42^{7}$ ends in a 7.

WMML
Meet #4
Feb. 1, 2022

Name ____________________

School ____________________

Algebra 1

1) There are 16 coins in a piggy bank. If the coins are all nickels and dimes and they total \$1.05, how many nickels are there?

1. 11

$$5n + 10(16 - n) = 105$$

$$-5n + 160 = 105$$

$$n = 11$$

2) Find all (x, y) such that $2\sqrt{x} + 4\sqrt{y} = 10$ and $2\sqrt{x} - 3\sqrt{y} = 3$.

2. $(9,1)$

Subtracting the second equation from the first gives $7\sqrt{y} = 7$, or $\sqrt{y} = 1$, so $y = 1$.

$2\sqrt{x} + 4(1) = 10$, so $\sqrt{x} = 3$ and $x = 9$.

3) Simplify the following expression:

$$\left(2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2}\right)^2$$

3. 4

$$\left(2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}} + \frac{1}{\sqrt{2} - 2}\frac{-\sqrt{2} - 2}{-\sqrt{2} - 2}\right)^2$$

$$= \left(2 + \sqrt{2} + \frac{2 - \sqrt{2}}{2} + \frac{-2 - \sqrt{2}}{2}\right)^2 = 2^2 = 4$$

WMML
Meet #4
Feb. 1, 2022

Name ____________________

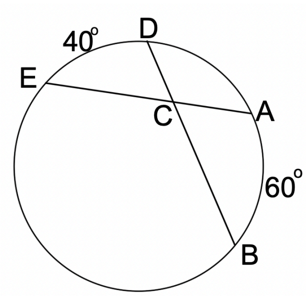
School ____________________

Geometry

130

1. ____________________

1) In the figure, if $\overset{\frown}{AB} = 60^\circ$ and $\overset{\frown}{DE} = 40^\circ$, then what is $\angle ACD$?



$$\angle ACB = \frac{60^\circ + 40^\circ}{2} = 50^\circ$$

$$\angle ACD = 180^\circ - 50^\circ = 130^\circ$$

15

2. ____________________

2) A 25-foot ladder is placed against a vertical wall. The foot of the ladder is 7 feet from the base of the wall. If the top of the ladder slips 4 feet, then how far will the foot slide?

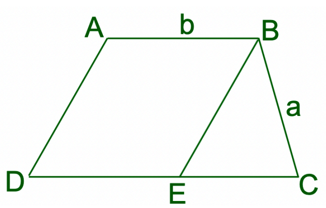
The top of the ladder is $\sqrt{25^2 - 7^2} = \sqrt{576} = 24$ feet up the wall. After it slides, it is 20 feet up the wall.

$$\sqrt{25^2 - 20^2} = \sqrt{225} = 15$$

a+b

3. ____________________

3) In the figure below, segments AB and CD are parallel, the measure of angle B is twice that of angle D, and the measures of segments CB and AB are a and b respectively. Find CD in terms of a and b.



$\angle ADE = \angle BEC = \angle EBC$, so $EC = BC = a$

$DE = AB = b$

$CD = CE + ED = a + b$

WMML
Meet #4
Feb. 1, 2022

Name ____________________

School ____________________

Algebra 2

1) How many integers satisfy $|x| + 1 \geq 3$ and $|x - 1| < 3$?

1. 2

First inequality: $|x| \geq 2$ so $x \leq -2$ or $x \geq 2$.

Second: $-3 < x - 1 < 3$ so $-2 < x < 4$

The only integers that satisfy both are 2 and 3, so there are 2 solutions.

2) Find the sum

$$\frac{1}{3+2\sqrt{2}} + \frac{1}{2\sqrt{2}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}+2} + \frac{1}{2+\sqrt{3}}$$

2. $3 - \sqrt{3}$

$$\frac{1}{\sqrt{9}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{3}}$$

Rationalizing each denominator gives the expression

$$\sqrt{9} - \sqrt{8} + \sqrt{8} - \sqrt{7} + \sqrt{7} - \sqrt{6} + \sqrt{6} - \sqrt{5} + \sqrt{5} - \sqrt{4} + \sqrt{4} - \sqrt{3}$$

$$= \sqrt{9} - \sqrt{3} = 3 - \sqrt{3}$$

3) Find $x^6 + \frac{1}{x^6}$ if $x + \frac{1}{x} = 3$.

3. 322

$x^2 + \frac{1}{x^2} + 2 = \left(x + \frac{1}{x}\right)^2 = 3^2 = 9$, so $x^2 + \frac{1}{x^2} = 7$.

$$\left(x^2 + \frac{1}{x^2}\right)^3 = x^6 + 3(x^2)^2\left(\frac{1}{x^2}\right) + 3(x^2)\left(\frac{1}{x^2}\right)^2 + \frac{1}{x^6}$$

$$7^3 = x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right)$$

$$343 = x^6 + \frac{1}{x^6} + 3(7) \quad \text{so} \quad x^6 + \frac{1}{x^6} = 343 - 21 = 322$$

WMML
Meet #4
Feb. 1, 2022

Name ______________________

School ____________________

Trigonometry and Complex Numbers

-2

1) Find the value of $\sec(1920°)$.

1.____________________

$$1920 = 120 + 360(5)$$

$$\sec(1920°) = \sec(120°) = \frac{1}{\cos(120°)} = \frac{1}{-1/2} = -2$$

3

2) What is the radius of a circle that is inscribed in a triangle with side lengths 8, 15 and 17?

2.____________________

$$r = \frac{\text{area of triangle}}{\text{semiperimeter of triangle}}$$

$$\text{area} = \frac{1}{2}(8)(15) = 60 \ \ (\text{since } 8^2 + 15^2 = 17^2)$$

$$\text{semiperimeter} = \frac{8 + 15 + 17}{2} = 20$$

$$r = \frac{60}{20} = 3$$

$2 + i$

3) If $f(z) = \dfrac{z+1}{z-1}$, then find $f^{2022}(2 + i)$.

3.____________________

$$f(2+i) = \frac{2+i+1}{2+i-1} = \frac{3+i}{1+i} \cdot \frac{1-i}{1-i} = \frac{4-2i}{2} = 2 - i$$

$$f^2(2+i) = f(2-i) = \frac{2-i+1}{2-i-1} = \frac{3-i}{1-i} \cdot \frac{1+i}{1+i} = \frac{4+2i}{2} = 2 + i$$

It keeps flipping back and forth, and $f^{2022}(2+i) = 2 + i$.

WMML
Meet #4
Feb. 1, 2022

Name ______________________

School ____________________

Precalculus

1) Find the product of all roots of the polynomial

$$x^3 + x^2 - 17x + 15.$$

1. -15

$$x^3 + x^2 - 17x + 15 = (x-1)(x-3)(x+5)$$

$$1(3)(-5) = -15$$

2) Find the equations of all asymptotes for the equation

$$\frac{(x+1)^2}{4} - \frac{(y-2)^2}{9} = 1.$$

2. $y - 2 = \pm \frac{3}{2}(x+1)$

Center is at $(-1,2)$, $a = 2$, and $b = 3$.

The asymptotes are defined by $y - 2 = \pm \frac{3}{2}(x+1)$.

3) Find the cosine of the angle between the vectors $(3 \quad 4 \quad 5)$ and $(-1 \quad 4 \quad 3)$.

3. $\frac{14\sqrt{13}}{65}$

$$\cos(\theta) = \frac{(3 \quad 4 \quad 5) \cdot (-1 \quad 4 \quad 3)}{||(3 \quad 4 \quad 5)|| \cdot ||(-1 \quad 4 \quad 3)||}$$

$$= \frac{3(-1) + 4(4) + 5(3)}{\sqrt{3^2+4^2+5^2} \cdot \sqrt{(-1)^2+4^2+3^2}} = \frac{28}{\sqrt{50} \cdot \sqrt{26}}$$

$$= \frac{28}{10\sqrt{13}} = \frac{14\sqrt{13}}{65}$$

WMML
Meet #4
Feb. 1, 2022

Name ______________________

School ____________________

Team Round

400

1. If $a \sharp b = a^b + b$, determine the value of $(4\sharp 5) - (5\sharp 4)$.

1.______________________

$$\left(4^5 + 5\right) - \left(5^4 + 4\right)$$

$$= (1024 + 5) - (625 + 4)$$

$$= 1029 - 629 = 400$$

$$b = \frac{-a}{2} + \frac{1}{2}$$

2. If $f(x) = 5x - 2$, $g(x) = ax + b$, and $f(g(x)) = g(f(x))$, find an expression for b in terms of a.

2.______________________

$$5(ax + b) - 2 = a(5x - 2) + b$$

$$5ax + 5b - 2 = 5ax - 2a + b$$

$$4b = -2a + 2$$

$$b = \frac{-a}{2} + \frac{1}{2}$$

768

3. How many cubes, each 3 inches on an edge, are needed to make a volume equal to that of a rectangular solid whose dimensions are 2 feet by 2 feet by 3 feet?

3.______________________

$$24 \times 24 \times 36 = 20{,}736$$

$$3 \times 3 \times 3 = 27$$

$$20{,}736 \div 27 = 768$$

WMML
Meet #4
Feb. 1, 2022

Name ____________________

School ____________________

4. Let x be an integer such that $-20 \leq x \leq 20$. If x is chosen at random, determine the probability that it will be a solution to both $|x-5| \geq 5$ and $x^2 \leq 196$.

4. $\frac{20}{41}$

There are 41 integers on the interval $-20 \leq x \leq 20$.

Since $x^2 \leq 196$, we have that $-14 \leq x \leq 14$.

For $|x-5| \geq 5$, we have that $x-5 \geq 5$ $(x \geq 10)$ or $x-5 \leq -5$ $(x \leq 0)$.

Both are true when

$x = -14, -13, -12, -11, -10, -9, -8, -7, -6, -5,$

$-4, -3, -2, -1, 0, 10, 11, 12, 13,$ and 14

5. If $F(x) = 3x^3 - 2x^2 + x - 3$, find $F(1+i)$.

5. $-8+3i$

$$F(1+i) = 3(1+i)^3 - 2(1+i)^2 + (1+i) - 3$$

$$= 3(-2+2i) - 2(2i) + (1+i) - 3$$

$$= (-6+6i) - 4i + (1+i) - 3 = -8+3i$$

6. If $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ b & 0 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 12 & 6 \end{bmatrix}$, determine a and b.

6. $a = -2, \quad b = 3$

$2a + 4b = 8$ $\qquad\qquad$ $-4a - 8b = -16$

$6a + 8b = 12$ $\qquad\qquad$ $6a + 8b = 12$

$$2a = -4 \quad \rightarrow \quad a = -2$$

$$2(-2) + 4b = 8 \quad \rightarrow \quad b = 3$$