WMML
Meet #1
Dec. 7, 2021

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School ____________________

Arithmetic and Number Theory

77

1) Let $p(n)$ be defined as the sum of all prime numbers from 1 to n, inclusive. Find $p(20)$.

1.____________________

$p(20) = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 = 77$

42

2) Let $q(n)$ be defined as the sum of all unique prime factors of n. Find $q(1776)$.

2.____________________

$1776 = 2^4 \cdot 3 \cdot 37$

$q(1776) = 2 + 3 + 37 = 42$

5

3) Let $p(n)$ and $q(n)$ be defined as above. Find $q(q(q(p(30))))$.

3.____________________

$p(30) = p(20) + 23 + 29 = 77 + 23 + 29 = 129 = 3 \cdot 43$

$q(p(30)) = q(129) = 3 + 43 = 46 = 2 \cdot 23$

$q(q(p(30))) = q(46) = 2 + 23 = 25 = 5^2$

$q(q(q(p(30)))) = q(25) = 5$

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Algebra 1

15

1) If x and y are inversely proportional and $x = 10$ when $y = 6$, what is x when $y = 4$?

1.____________________

$x \cdot y = 10 \cdot 6 = 60$

$x \cdot y = x \cdot 4 = 60$

$x = 15$

$26\frac{2}{3}$ mph

2) Chad drives to his mother's house, which is 40 miles away, and then drives back. On the way there he drives 40 miles per hour, but on the way back he drives only 20 miles per hour. What is his average speed for the whole trip?

2.____________________

He drives a total of 80 miles. It takes 1 hour to get there, and 2 hours to get back. His average speed is $\frac{80}{3} = 26\frac{2}{3}$ mph.

12

3) Evaluate the following expression:

3.____________________

$$\frac{\left(4^{\frac{2}{3}}\right)\left(2^{\frac{1}{6}}\right)\left(3^{\frac{3}{2}}\right)}{\left(2^{-\frac{1}{2}}\right)\left(3^{\frac{1}{2}}\right)}$$

$$\frac{\left(2^{\frac{4}{3}}\right)\left(2^{\frac{1}{6}}\right)\left(3^{\frac{3}{2}}\right)}{\left(2^{-\frac{1}{2}}\right)\left(3^{\frac{1}{2}}\right)} = 2^{\frac{4}{3}+\frac{1}{6}-(-\frac{1}{2})} \cdot 3^{\frac{3}{2}-\frac{1}{2}} = 2^2 \cdot 3 = 12$$

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Geometry

1) The point $A(2,4)$ is translated 5 units down, reflected across the x-axis, reflected across the y-axis, rotated 90° clockwise around the origin, and dilated by a factor of 3 (centered at the origin), in that order. What are the coordinates of the image after all the transformations?

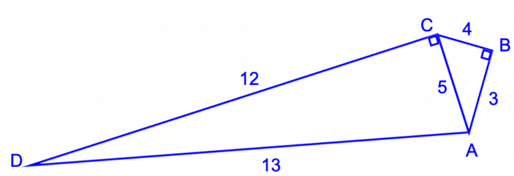
1. (3,6)

$$(2,4) \longrightarrow (2,-1) \longrightarrow (2,1) \longrightarrow (-2,1) \longrightarrow (1,2) \longrightarrow (3,6)$$

2) Sides AB, BC, CD, and DA of convex quadrilateral $ABCD$ have lengths 3, 4, 12, and 13, respectively; and $\angle CBA$ is a right angle. What is the area of the quadrilateral?

2. 36

Triangle ABC is a right triangle, so $AC = 5$. The sides of triangle ACD are 5, 12, and 13, making it a right triangle. Therefore the area is $A = \frac{1}{2}(3)(4) + \frac{1}{2}(5)(12) = 36$.



3) A circle is inscribed in a triangle with sides of lengths 8, 13, and 17. Let the segments of the side of length 8 made by a point of tangency be r and s, with $r < s$. Find the ratio $r : s$.

3. 2:6

$$r = \frac{8+13-17}{2} = 2$$

$$s = \frac{8+17-13}{2} = 6$$

$$r : s = 2 : 6$$

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Algebra 2

1) The graphs of the equations $x + 3y = 6$ and $kx + 2y = 12$ are perpendicular. What is the value of k?

1. -6

$y = -\frac{1}{3}x + 2$ has a slope of $-\frac{1}{3}$.

$y = -\frac{k}{2}x + 6$ has a slope of $-\frac{k}{2}$.

Since they are perpendicular: $-\frac{k}{2} = 3 \longrightarrow k = -6$

2) Find all x such that $1 + \frac{x+3}{x-2} = \frac{3x-3}{6-x}$.

2. 0 and 4

$$(6-x)(x-2)\left(1 + \frac{x+3}{x-2}\right) = (6-x)(x-2)\left(\frac{3x-3}{6-x}\right)$$

$$(6-x)(x-2) + (6-x)(x-2)\left(\frac{x+3}{x-2}\right) = (x-2)(3x-3)$$

$$-2x^2 + 11x + 6 = 3x^2 - 9x + 6$$

$$5x^2 - 20x = 0 \longrightarrow x = 0 \quad \text{and} \quad x = 4$$

3) Find all values of m which will make $x + 2$ a factor of $x^3 + 3m^2x^2 + mx + 4$.

3. $m = \frac{2}{3}$ and $m = -\frac{1}{2}$

$$(-2)^3 + 3m^2(-2)^2 + m(-2) + 4 = 0$$

$$12m^2 - 2m - 4 = 0$$

$$(6m-4)(2m+1) = 0$$

$$m = \frac{2}{3} \text{ and } m = -\frac{1}{2}$$

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Trigonometry and Complex Numbers

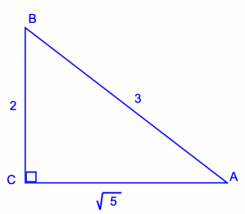
1) Find the value of $\left|\dfrac{7-24i}{4+3i}\right|$.

1.___5___

$$\frac{|7-24i|}{|4+3i|}=\frac{\sqrt{7^2+24^2}}{\sqrt{4^2+3^2}}=\frac{\sqrt{625}}{\sqrt{25}}=5$$

2) Triangle ABC has a right angle at C. If $\sin(A)=\dfrac{2}{3}$, then find the exact value of $\sin(B)\cdot\cos(A)\cdot\tan^2(B)$.

2.___$\dfrac{25}{36}$___



$$\sin(B)=\cos(A)=\frac{\sqrt{5}}{3}$$

$$\tan(B)=\frac{\sqrt{5}}{2}$$

$$\sin(B)\cdot\cos(A)\cdot\tan(B)=\frac{\sqrt{5}}{3}\cdot\frac{\sqrt{5}}{3}\cdot\left(\frac{\sqrt{5}}{2}\right)^2=\frac{25}{36}$$

3) If the 6 solutions to $x^6=-64$ are written in the form $a+bi$, where a and b are real, then find the product of those solutions with $a>0$.

3.___4___

If we write -64 as $64e^{i\pi}$, then one sixth root will be $64^{1/6}e^{i\pi/6}=2e^{i\pi/6}$. All other roots differ by an angle of $\dfrac{2\pi}{6}=\dfrac{\pi}{3}$.

The only roots that have a positive value of a are $2e^{i\pi/6}$ and $2e^{-i\pi/6}$. Their product is $2e^{i\pi/6}\cdot 2e^{-i\pi/6}=4\cdot e^0=4$.

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Precalculus

$\left(4\sqrt{2}, \frac{7\pi}{4}\right)$

1) Find the polar coordinate representation of the center of the circle defined by the equation $(x-4)^2 + (y+4)^2 = 37$.

1.____________________

Center is at $(4, -4)$.

$r = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$

$\theta = \frac{7\pi}{4}$

$(r, \theta) = \left(4\sqrt{2}, \frac{7\pi}{4}\right)$

$k = \frac{5}{3}$ and $k = 1$

2) The line through $(3, \frac{8}{3}k)$ and $(4, k^2 + 2)$ is perpendicular to the line $3x + y = 2021$. Find all possible values of k.

2.____________________

Slope through the given points is $k^2 - \frac{8}{3}k + 2$.

Slope of given line is -3, so $k^2 - \frac{8}{3}k + 2 = \frac{1}{3}$.

$$3k^2 - 8k + 5 = (3k-5)(k-1) = 0$$

$k = \frac{5}{3}$ and $k = 1$

29,120

3) In the expansion of $(xy - 2y^{-3})^{16}$, find the coefficient of the term that does not contain y.

3.____________________

By Binomial Theorem, a general term of the expansion looks like

$$\binom{16}{k}(xy)^k(-2y^{-3})^{16-k} = \binom{16}{k}(-2)^{16-k}x^k y^{4k-48}.$$

There is no power of y when $4k - 48 = 0$, so when $k = 12$.

$$\binom{16}{12}(-2)^{16-12} = \frac{16!}{12!4!} \cdot 16 = \frac{16^2 \cdot 15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2} = 29{,}120$$