WMML
Meet #1
Nov. 9, 2021

Name _____

School _____

Arithmetic and Number Theory

$202.10

1) How much, in dollars, is $2{,}021{,}000\,\%$ of a penny?

1._____

$$\frac{2{,}021{,}000}{100}(0.01) = \frac{2{,}021{,}000}{10{,}000} = 202.1$$

63

2) Starting with 1, at most how many consecutive positive integers can be added before the sum exceeds 2021?

2._____

$$\frac{n(n+1)}{2} \le 2021 \quad \rightarrow \quad n(n+1) \le 4042$$

$63 \times 64 = 4{,}032$ and $64 \times 65 = 4160$

64 and 15,625

3) Find two numbers whose product is $1{,}000{,}000$ such that neither number contains a zero.

3._____

$1{,}000{,}000 = 10^6 = 2^6 \cdot 5^6$

Neither factor can contain $2 \cdot 5$, so we must have $2^6 = 64$ and $5^6 = 15{,}625$ as the two factors.

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Algebra 1

1) Solve the following equation for x:

$$|4x - 5| = 2x + 1$$

$x = 3, \quad x = \frac{2}{3}$

1._____

$4x - 5 = 2x + 1 \quad \rightarrow \quad x = 3$

$-(4x - 5) = 2x + 1 \quad \rightarrow \quad x = \frac{2}{3}$

2) For what values of n is $n^{-3} = \left(\frac{1}{n}\right)^5$?

$1, -1$

2._____

$n^{-3} \times n^5 = n^{-5} \times n^5$

$n^2 = 1 \quad \rightarrow \quad n = \pm 1$

3) Bryan has a part-time job delivering packages. He is paid a flat rate of $9.50 per hour. Caleb works at a competitor that pays its employees $2 per hour plus $3 per delivery. How many deliveries would Caleb have to make in 40 hours to earn the same pay as Bryan for a a 40 hour work week?

100

3._____

$9.5(40) = 2(40) + 3x$

$300 = 3x$

$x = 100$

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Geometry

100

1) How many pipes with inside diameter 1 inch are needed to carry the same amount of water as one pipe with inside diameter 10 inches?

1._____

$$A_1 = \pi\left(\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)\pi$$

$$A_2 = \pi(5)^2 = 25\pi$$

$$25\pi \div \left(\frac{1}{4}\right)\pi = 100$$

40,000%

2) The length of each side of a triangle is increased by $2{,}000\,\%$. By what percent does the area of the triangle increase?

2._____

$$A_1 = \frac{1}{2}bh$$

$$A_2 = \frac{1}{2}(20b)(20h) = 400\left(\frac{1}{2}bh\right)$$

There is a $40{,}000\,\%$ increase in the area.

24

3) An equilateral triangle has perimeter P and area A. If $A = 2\sqrt{3}P$, then what is the side length of the triangle?

3._____

Side length $= \frac{P}{3}$.

$$A = \frac{1}{2}\left(\frac{P}{3}\right)\left(\frac{\sqrt{3}P}{6}\right) = 2\sqrt{3}P$$

$$P^2 = 72P \quad \rightarrow \quad P(P-72) = 0 \quad \rightarrow \quad P = 72$$

$$\text{Side length} = \frac{72}{3} = 24$$

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Algebra 2

64

1) The vertex of the parabola $y = x^2 - 16x + k$ is on the x-axis. What is the value of k?

1._____

$$\frac{-16}{2} = -8$$

$$y = (x-8)^2 = x^2 - 16x + 64$$

$2,700

2) Years ago Jack purchased shares of Tesla for a total of $3,000. The value of his investment grew by 350% and he wants to give the shares to his 4 children in the ratio of 3:3:2:2. How much money is the smallest gift worth?

2._____

$$3{,}000(4.5) = 13{,}500$$

$$3x + 3x + 2x + 2x = 13{,}500$$

$$x = \frac{13{,}500}{10} = 1{,}350$$

-512

3) If $(x-3y)^3(x+3y)^3$ is written as a polynomial in x and y, what is the sum of its coefficients?

3._____

$$(x-3y)^3(x+3y)^3 = (x^2-9y^2)^3$$

$$(x^2-9y^2)^3 = 1x^6 + 3(x^4)(-9y^2) + 3(x^2)(-9y^2)^2 + 1(-9y^2)^3$$

$$= x^6 - 27x^4y^2 + 243x^2y^4 - 729y^3$$

$$1 - 27 + 243 - 729 = -512$$

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Trigonometry and Complex Numbers

1) Solve for all x where $0 \le x < 2\pi$.

π

1._____

$$\sin(x-\pi) = \cot\left(\frac{\pi}{6}\right) - 2\cos\left(\frac{\pi}{6}\right)$$

$$\sin(x-\pi) = \sqrt{3} - 2\left(\frac{\sqrt{3}}{2}\right) = 0$$

$$x = \pi$$

2) Simplify $\left(i^{2021} + i^{2022} + i^{2023} + i^{2024}\right)^{2021}$.

0

2._____

$$\left(i^1 + i^2 + i^3 + i^4\right)^{2021} = (i - 1 - i + 1)^{2021} = 0^{2021} = 0$$

3) Find all possible values of $\angle PQR$ if $PQ = 12$, $QR = 4\sqrt{3}$, and the area of $\triangle PQR$ is $12\sqrt{6}$.

45° or 135°

3._____

$$A = \frac{1}{2}PQ \cdot QR \sin\left(\angle PQR\right) = \frac{1}{2} \cdot 12 \cdot 4\sqrt{3} \sin\left(\angle PQR\right)$$

$$= 24\sqrt{3} \sin\left(\angle PQR\right)$$

$$\sin\left(\angle PQR\right) = \frac{12\sqrt{6}}{24\sqrt{3}} = \frac{\sqrt{2}}{2}$$

$$\sin\left(\angle PQR\right) = 45^\circ \text{ or } 135^\circ$$

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Precalculus

1) What is the slope of the line that goes through the point $(-3,2)$ and the intersection of the lines $y = 2x + 1$ and $3x + y = 11$?

$\frac{3}{5}$

1._____

$3x + (2x + 1) = 11 \quad \rightarrow \quad x = 2$

$y = 2(2) + 1 = 5$

$m = \frac{5 - 2}{2 - (-3)} = \frac{3}{5}$

2) The natural numbers from 1 to 2021 are placed into a bag. If one number is drawn randomly from the bag, what is the probability that it is not a multiple of 2 or 3?

$\frac{674}{2021}$

2._____

Multiples of 2: $\frac{2021}{2} = 1010r1$

Multiples of 3: $\frac{2021}{3} = 673r2$

Multiples of 2 and 3: $\frac{2021}{6} = 336r5$

$1 - \frac{1010 + 673 - 336}{2021} = \frac{674}{2021}$

3) Find the value of k such that the graphs of $(x + 2)^2 + (y - 5)^2 = k$ and $(x - 4)^2 + (y - 1)^2 = k$ have only one point of intersection.

13

3._____

$\sqrt{(4 + 2)^2 + (1 - 5)^2} = \sqrt{52} = 2\sqrt{13}$

The radius if each circle must be $r = \frac{2\sqrt{13}}{2} = \sqrt{13}.$

$k = r^2 = 13.$

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Team Round

503

1. Starting from right, how many zeros are there before the first non-zero digit after expanding the factorial expression 2021!?

1._____

We need to find the number of 5's in the prime factorization of 2021!

$$\frac{2021}{5} = 404r1\ , \frac{2021}{25} = 80r21\ , \frac{2021}{125} = 16r21$$

$$\frac{2021}{625} = 3r146 \quad \rightarrow \quad 404 + 80 + 16 + 3 = 503$$

43

2. How many numbers x in the set $\{1, 2, 3, \ldots, 2021\}$ are there such that $x^2 + x^3$ equals the square of an integer?

2._____

$$x^2 + x^3 = x^2(1 + x)$$

Since x^2 is a square, we need $1 + x$ to also be a square. The first such value is when $x = 3$ since $3 + 1 = 4 = 2^2$. The largest such value is $x = 1935$ since $1{,}935 + 1 = 1{,}936 = 44^2$. Therefore there are 43 such values of x.

126°

3. Given the regular decagon $ABCDEFGHIJ$, find the measure of $\angle HIA$.

3._____

$$m\angle HIJ = \frac{8(180)}{10} = 144$$

$$m\angle JIA = \frac{180 - 144}{2} = 18$$

$$m\angle HIA = 144 - 18 = 126$$

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2021

4. Three whole numbers, when added together two at a time, have sums of 1202, 2223, and 3021. Determine the value of the largest of the original three numbers.

4._____

$x + y = 1202$ $\quad y = 1202 - x$ $\quad 2z = 4042$

$x + z = 2223$ $\quad 1202 - x + z = 3021$ $\quad z = 2021$

$y + z = 3021$ $\quad -x + z = 1819$

$1 + a$

5. If $\left(\sin(x) - \cos(x)\right)^2 = a^2$, express $\dfrac{\sin(2x)}{1-a}$ in simplest form in terms of a.

5._____

$\sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x) = a^2$

$1 - \sin(2x) = a^2 \quad \rightarrow \quad \sin(2x) = 1 - a^2$

$\dfrac{\sin(2x)}{1-a} = \dfrac{1-a^2}{1-a} = 1 + a$

$\begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix}$

6. Let $\mathbf{v} = \begin{pmatrix} 4\sqrt{3} \\ 4 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ be vectors. Find the projection of **v** onto **w**.

6._____

$proj_w v = \dfrac{v \cdot w}{||w||^2} w = \dfrac{(4\sqrt{3})(1) + (4)(\sqrt{3})}{2^2} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix}$