WMML
Meet #1
Oct. 5, 2021

Name _____

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Arithmetic and Number Theory

1) What is the greatest common factor of 1005 and 2021?

1. 1

$$1005 = 3 \cdot 5 \cdot 67 \quad \text{and} \quad 2021 = 43 \cdot 47$$

$$\text{Therefore } \gcf(1005, 2021) = 1$$

2) The sum of 10 consecutive even integers is 450; find the largest integer.

2. 54

$$x + (x+2) + (x+4) + \cdots + (x+18) = 450$$

$$10x + 90 = 450$$

$$x = 36$$

$$x + 18 = 36 + 18 = 54$$

3) The number 2021_8 is written in base 8 notation. What is this number in base 10?

3. 1041

$$2021_8 = \left(2(8)^3 + 0(8)^2 + 2(8)^1 + 2(8)^0\right)_{10}$$

$$= \left(1024 + 0 + 16 + 1\right)_{10} = 1041$$

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Algebra 1

1) You fill your car's gas tank when it is $\frac{1}{2}$ full of gas. The next week, you fill the tank a second time when it is $\frac{1}{4}$ full of gas. If you buy a total of $18\frac{1}{2}$ gallons of gas on these two days, then how many gallons does the tank hold?

$$\frac{1}{2}x + \frac{3}{4}x = 18\frac{1}{2}$$

$$\frac{5}{4}x = \frac{37}{2}$$

$$x = \frac{74}{5} = 14\frac{4}{5} \text{ gallons}$$

$14\frac{4}{5}$

1._____

2) Line k contains the point $(2, -2)$ and has the same x-intercept as the line $y + 9 = 3(x - 4)$. Write the equation for line k in slope intercept form.

$$0 + 9 = 3x - 12 \quad \Longrightarrow \quad x = 7$$

$$m = \frac{0 - (-2)}{7 - 2} = \frac{2}{5}$$

$$y - 0 = \frac{2}{5}(x - 7) \quad \Longrightarrow \quad y = \frac{2}{5}x - \frac{14}{5}$$

$y = \frac{2}{5}x - \frac{14}{5}$

2._____

3) You have 58 coins in your pocket that are either quarters or nickels. They total $10.30. How many of each coin do you have?

$$x + y = 58 \quad \text{and} \quad 0.25x + 0.05y = 10.30$$

$$0.25(58 - y) + 0.05y = 10.30$$

$$14.5 - 0.2y = 10.3$$

$$y = 21 \text{ nickels} \quad \text{and} \quad x = 58 - 21 = 37 \text{ quarters}$$

21 nickels

3._____37 quarters_____

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Geometry

1) One right circular cone is set inside a larger right circular cone. The cones share the same axis, the same vertex, and the same height. Find the volume of the space between the cones if the diameter of the cones are 6 inches and 9 inches, and they both have a height of 5 inches.

$$\left(\frac{1}{3}\right)(\pi)\left(\frac{9}{2}\right)^2(5) - \left(\frac{1}{3}\right)(\pi)(3)^2(5)$$

$$= \frac{135\pi}{4} - 15\pi = \frac{75\pi}{4}$$

1. $\frac{75\pi}{4}$

2) A rectangular prism has a length of $x + 7$, width of x, and height of x. It has the same surface area as an 8-in cube. What is the value of x?

$$6(8^2) = 384$$

$$2x^2 + 4(x)(x+7) = 384$$

$$6x^2 + 28x - 384 = 0$$

$$x = \frac{-28 \pm \sqrt{28^2 - 4(6)(-384)}}{12} = \frac{-28 \pm \sqrt{10000}}{12} = \frac{-28 \pm 100}{12}$$

$$x = \frac{-28 + 100}{12} = 6$$

2. 6

3) The sides of a triangle are x, $x - 2$, and $x + 2$. The area of the triangle is $\frac{\sqrt{27}}{2}$. Determine the value of x.

$$s = \frac{x + x - 2 + x + 2}{2} = \frac{3x}{2}$$

$$\frac{27}{4} = \frac{3x}{2}\left(\frac{3x}{2} - x\right)\left(\frac{3x}{2} - x + 2\right)\left(\frac{3x}{2} - x - 2\right)$$

$$\frac{27}{4} = \frac{3x^2}{4}\left(\frac{x^2}{4} - 4\right) \implies \frac{27}{4} = \frac{3x^4}{16} - 3x^2$$

$$3x^4 - 48x^2 - 108 = 0 \implies x^4 - 16x^2 - 36 = 0$$

$$(x^2 - 18)(x^2 + 2) \implies x^2 = 18 \implies x = 3\sqrt{2}$$

3. $3\sqrt{2}$

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Algebra 2

1) Find all solutions to the equation $\sqrt{2x+1} - \sqrt{x} = 1$.

1. 0, 4

$$\sqrt{2x+1} = \sqrt{x} + 1$$
$$2x + 1 = x + 2\sqrt{x} + 1$$
$$x = 2\sqrt{x}$$
$$x^2 = 4x$$
$$x = 0 \quad \text{or} \quad x = 4$$

2) A circle is centered at the point $(4, -1)$ and passes through the point $(-1,2)$. What is the equation of the circle?

$(x-4)^2 + (y+1)^2 = 34$

2. _____

$$\text{Radius} = \sqrt{(4+1)^2 + (-1-2)^2} = \sqrt{34}$$

$$(x-4)^2 + (y+1)^2 = 34$$

3) You have 88 coins in your pocket that are either quarters, dimes, or nickels. They total $10, and there are twice as many dimes as nickels. How many quarters do you have?

3. 16

$$\begin{array}{c} x + y + z = 88 \\ .05x + .1y + .25z = 10 \\ y = 2x \end{array} \implies \begin{array}{c} 3x + z = 88 \\ .25x + .25z = 10 \end{array}$$

$$.25x + .25(88 - 3x) = 10$$

$$22 - .5x = 10 \implies x = 24$$

$$z = 88 - 3(24) = 16$$

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Trigonometry and Complex Numbers

1) Find the area of $\triangle ABC$ if $AB = 4BC = 12$ and $\angle ABC = 30^\circ$.

1._____9_____

We see that $BC = \frac{12}{4} = 3$, so the area of $\triangle ABC$ is

$$\frac{1}{2} AB \cdot BC \cdot \sin(\angle ABC) = \frac{1}{2} \cdot 12 \cdot 3 \cdot \frac{1}{2} = 9.$$

2) If θ is the acute angle formed by the x-axis and the graph of the line $\pi x + 3y = 7$, then find the value of $\tan(\theta)$.

$\frac{\pi}{3}$

2._____

The line has x-intercept of $\left(\frac{7}{\pi}, 0\right)$ and a y-intercept of $\left(0, \frac{7}{3}\right)$.

So we have $\tan(\theta) = \frac{7/3}{7/\pi} = \frac{\pi}{3}$.

3) Let $w = 3 - 4i$ and $z = 2 + 5i$. Express the following as a single complex number:

$-69 - 158i$

3._____

$$|w| \cdot \frac{w^2 z - w z^2}{z + w + 4i}.$$

$$\sqrt{3^2 + (-4)^2} \cdot \frac{(3-4i)^2(2+5i) - (3-4i)(2+5i)^2}{(3-4i) + (2+5i) + 4i}$$

$$= 5 \cdot \frac{(-7-24i)(2+5i) - (3-4i)(-21+20i)}{5+5i}$$

$$= \frac{(106-83i) - (17+144i)}{1+i} = \frac{(89-227i)(1-i)}{(1+i)(1-i)}$$

$$= \frac{-138-316i}{2} = -69-158i$$

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Precalculus

1) A circle is defined by equation $x^2 + y^2 - 2x - 8y + 10 = 0$. Find the distance between the origin and the center of this circle.

$\sqrt{17}$

1._____

$$(x^2 - 2x + 1) + (y^2 - 8y + 16) = -10 + 1 + 16$$

$$(x-1)^2 + (y-4)^2 = 7 \implies \text{Center is at } (1,4).$$

$$\text{Distance to origin } = \sqrt{1^2 + 4^2} = \sqrt{17}$$

2) Let $A(-9,3)$, $B(2,0)$, and $C(0,y)$ be three points. Find all values of y such that $\angle ABC = 90°$.

2._____6, -3_____

$$\frac{y-3}{0-(-9)} = \frac{y-3}{9} \quad \text{and} \quad \frac{y-0}{0-2} = \frac{y}{-2}$$

$$\frac{y-3}{9} = \frac{2}{y}$$

$$y^2 - 3y = 18 \implies y^2 - 3y - 18 = 0$$

$$(y-6)(y+3) = 0 \implies y = 6 \text{ and } y = -3$$

3) Let $f(x) = \sqrt{2^{3x}}$. Find the value of k if $f(x+k) = 8f(x)$. Write your answer in the form of $\log_b(a)$.

$\log_2(4)$

3._____

$$f(x) = 2^{\frac{3x}{2}}$$

$$2^{\frac{3(x+k)}{2}} = 8 \cdot 2^{\frac{3x}{2}}$$

$$\frac{2^{\frac{3x}{2}+\frac{3k}{2}}}{2^{\frac{3x}{2}}} = 8$$

$$2^{\frac{3k}{2}} = 8$$

$$\log_2(8) = \frac{3k}{2}$$

$$k = \frac{2}{3}\log_2(8) = \log_2(4)$$