WMML
Meet #3
Dec. 3, 2019

Name ______________________

School _____________________

Arithmetic and Number Theory

1) Find the largest integer less than 80 that leaves a remainder of 3 when divided by 6.

1._________75___________

$$n = 6x + 3 < 80$$
$$x < \frac{77}{6} = 12\frac{1}{6}$$

We have that $x = 12$, and so $n = 75$.

2) What two-digit number is three times the sum of its digits?

2.__________27__________

$$10A + B = 3(A + B)$$
$$7A = 2B$$
$$A = \frac{2}{7}B$$

Since A and B are integers between 0 and 9 we must have $A = 2$ and $B = 7$, making the answer 27.

3) If n has exactly 7 positive divisors, how many positive divisors does n^2 have?

3._________13___________

The number of positive divisors is equal to the product of one more than each exponent in the prime factorization. Since there are 7 positive divisors we know that $n = p^6$ for some prime number p. So $n^2 = p^{12}$, and there are $12 + 1 = 13$ positive divisors.

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Algebra 1

1) Jeff is 4 times older than his daughter. Five years ago he was 9 times older than his daughter. How old is his daughter?

1.__________8__________

$$4x - 5 = 9(x - 5)$$
$$x = 8$$

2) Let $f(x) = a^2x^2 + \frac{5}{2}ax + 3$ and $f(2) = 2$. Find all possible values of the constant a.

2.____-1/4 and -1________

$$2 = 4a^2 + 5a + 3$$
$$4a^2 + 5a + 1 = 0$$
$$(4a + 1)(a + 1) = 0$$
$$a = -\frac{1}{4} \text{ or } a = -1$$

3) A tennis player computes her “win ratio” by dividing the number of matches she has won by the total number of matches she has played. At the start of the weekend her win ratio was exactly .500. During the weekend she played 4 matches, winning three and losing 1. At the end of the weekend her win ratio is greater than .503. What is the greatest number of matches she could have won before the weekend began?

3.__________164__________

$$\frac{n + 3}{2n + 4} > 0.503$$
$$n + 3 > 1.006n + 2.012$$
$$0.006n < 0.988$$
$$n < 164\frac{2}{3}$$

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Geometry

1) The perimeter of an isosceles triangle is 38 centimeters and two sides of the triangle are whole numbers in the ratio $3:8$. What is the number of centimeters in the length of the shortest side?

1.__________6__________

$3x, 3x, 8x$ is not possible since $3x + 3x = 6x < 8x$

$$3x + 8x + 8x = 19x = 38$$
$$x = 2$$

Side lengths are 6, 16, and 16.

2) Find the volume of a cube, given that the greatest distance between any two vertices is 6.

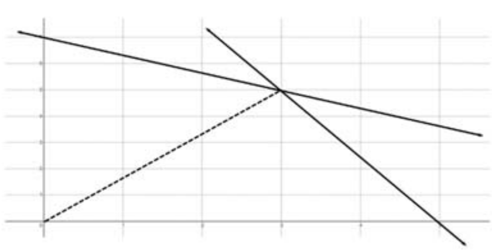
2.__________$24\sqrt{3}$__________

Let s equal the side length of the cube. Then we have

$$6 = \sqrt{\left(\sqrt{2}s\right)^2 + s^2} = \sqrt{3}s$$
$$s = 2\sqrt{3}$$
$$s^3 = \left(2\sqrt{3}\right)^3 = 24\sqrt{3}$$

3) Find the area of the region bounded by the lines $2x + 3y = 21$ and $5x + 2y = 25$ and the coordinate axes.

3.__________23__________



$$\frac{1}{2}(7)(3) = \frac{21}{2}$$
$$\frac{1}{2}(5)(5) = \frac{25}{2}$$
$$\frac{21}{2} + \frac{25}{2} = 23$$

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Algebra 2

1) One root of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ is $x = 1$. What is the other root in terms of a, b, and c?

1. $\dfrac{c(a-b)}{a(b-c)}$

$$(x-1)\big(a(b-c)x - c(a-b)\big) = 0$$

$$x = \frac{c(a-b)}{a(b-c)}$$

2) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$.

Find the value of $ABB^{-1}BAA^{-1}$.

2. $\begin{bmatrix} 7 & 1 \\ 16 & 3 \end{bmatrix}$

$$ABB^{-1}BAA^{-1} = AB$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1(3)+2(2) & 1(-1)+2(1) \\ 2(3)+5(2) & 2(-1)+5(1) \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 16 & 3 \end{bmatrix}$$

3) Simplify the sum $\sqrt[3]{18+5\sqrt{13}} + \sqrt[3]{18-5\sqrt{13}}$.

3. 3

$$x = \alpha + \beta = \sqrt[3]{18+5\sqrt{13}} + \sqrt[3]{18-5\sqrt{13}}.$$

$$x^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 = 18 + 5\sqrt{13} + 3\alpha\beta(\alpha+\beta) + 18 - 5\sqrt{13}$$
$$= 36 + 3x\sqrt[3]{18+5\sqrt{13}}\sqrt[3]{18-5\sqrt{13}} = 36 + 3x\sqrt[3]{(18+5\sqrt{13})(18-5\sqrt{13})}$$
$$= 36 + 3x\sqrt[3]{18^2 - 5^2\cdot 13} = 36 + 3x\sqrt[3]{-1} = 36 - 3x$$

$$x^3 + 3x - 36 = (x-3)(x^2+3x+12) = 0$$
$$x = 3$$

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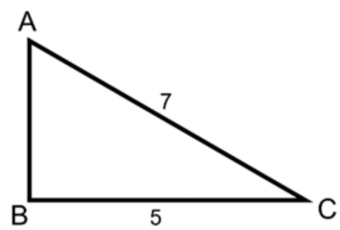
Trigonometry and Complex Numbers

1) In triangle ABC, we have $\angle B = 90°$ and $\sin(A) = \frac{5}{7}$. Find $\tan(C)$.

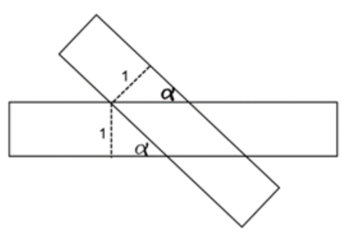
1. $\frac{2\sqrt{6}}{5}$



$$AB = \sqrt{7^2 - 5^2} = 2\sqrt{6}$$
$$\tan(C) = \frac{2\sqrt{6}}{5}$$

2) Two strips of width 1 overlap at an angle of α as shown. Find the area of the overlapping section in terms of α.

2. $\frac{1}{\sin(\alpha)}$



Height of parallelogram: 1

Length of parallelogram:

$$\frac{1}{\sin(\alpha)}$$

3) Find all complex numbers z such that $z^2 = 21 - 20i$.

3. -5+2i, 5-2i

$$z^2 = (a + bi)^2 = a^2 - b^2 + 2abi$$
$$2ab = -20, \text{ so } b = -\frac{10}{a} \text{ and } a^2 - b^2 = a^2 - \frac{100}{a^2} = 21$$
$$a^4 - 21a^2 - 100 = (a^2 - 25)(a^2 + 4) = 0$$
$$a = -5 \text{ or } a = 5$$

Since $b = -10/a$, we have that $b = 2$ and $b = -2$, respectively.

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Precalculus

1) Let P be the point that has polar coordinates $(8, 60°)$. Find the rectangular coordinates for the point P.

1. $(4, 4\sqrt{3})$

$$x = 8\cos(60°) = \frac{8}{2} = 4$$
$$y = 8\sin(60°) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$$

2) A hyperbola centered at the origin has one vertex at $(5,0)$ and one focus at $(-13,0)$. Find an equation whose graph is this hyperbola.

2. $\frac{x^2}{25} - \frac{y^2}{144} = 1$

Since $(5,0)$ is a vertex we have $a = 5$. Since $(-13,0)$ is a focus, we have $c = 13 = \sqrt{a^2 + b^2}$.

$$a^2 + b^2 = 169$$
$$b = 12$$

The equation is then $\frac{x^2}{5^2} - \frac{y^2}{12^2} = 1$

3) What are the rectangular coordinates of the point that results when $(-5\sqrt{2}, 5\sqrt{6})$ is rotated $\frac{\pi}{3}$ radians counter-clockwise about the origin?

3. $(-10\sqrt{2}, 0)$

$r^2 = x^2 + y^2 = 50 + 150 = 200$, so $r = 10\sqrt{2}$.

$\tan(\theta) = \frac{y}{x} = \frac{5\sqrt{6}}{-5\sqrt{2}} = -\sqrt{3}$, so $\theta = \frac{2\pi}{3}$.

Rotating the point $(10\sqrt{2}, \frac{2\pi}{3})$ gives the point $(10\sqrt{2}, \pi)$.

Converting back into rectangular coordinates gives $(-10\sqrt{2}, 0)$.