Name _____

School _____

Arithmetic and Number Theory

1) How many positive integer divisors of 30! are prime?

1._____

2) A palindrome is an integer that reads the same forward as it does backward, such as 12321. Find the total number of ten-digit palindromes.

2._____

3) How many positive integer divisors of 5400 are **not** multiples of any perfect square greater than 1?

3._____

Algebra 1

1) Find all ordered pairs (a, b) such that $3a - 2b = -8$ and $5a + 4b = 5$.

1._____

2) Let x and y be real numbers satisfying $\frac{2}{x} = \frac{y}{3} = \frac{x}{y}$.

Determine x^3.

2._____

3) Find all pairs of real numbers (a, b) such that $(x - a)^2 + (2x - b)^2 = (x - 3)^2 + (2x)^2$ for all x.

3._____

Geometry

1._____

1) $ABCDEFGH$ is a right rectangular prism with volume 672 in^3.
If $AB = 6\ in$ and $BC = 8\ in$, then what is length of the third
dimension?

2._____

2) Base $IJKL$ of right prism $IJKLMNOP$ is a parallelogram with
$IJ = 8, JK = 6$, and $\angle IJK = 60°$. Given that the height of the
prism is 9, find the surface area of $IJKLMNOP$.

3._____

3) $QRSTUVWX$ is a cube. How many different planes pass through
at least three vertices of $QRSTUVWX$?

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Algebra 2

1) Solve the inequality $x^2 - 12x + 32 \leq 0$.

1._____

2) The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has roots 1, 3, 5, and 7. Determine $a + b + c + d$.

2._____

3) There is a unique polynomial $P(x)$ of the form

$$P(x) = 7x^7 + bx^6 + cx^5 + \cdots + gx + h$$

such that $P(1) = 1, P(2) = 2, \ldots, P(7) = 7$. Find $P(0)$.

3._____

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Trigonometry and Complex Numbers
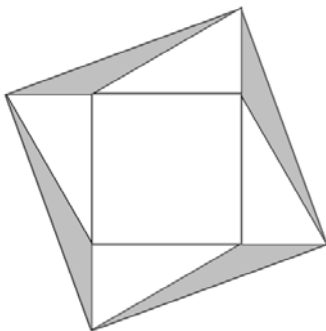
1) Evaluate $\cos(225°) + \sin(225°)$.

1._____

2) The complex number z satisfies $z + |z| = 2 + 8i$. What is $|z|$?

2._____

3) Four congruent 30-60-90 triangles are constructed on the sides of a square as shown below. The hypotenuse of each of these triangles has length 2. The outer vertices of these triangles are connected to form quadrilateral $ABCD$. What fraction of $ABCD$ is shaded?

3._____

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Precalculus

1) Determine the value of

$$\sum_{k=1}^{8}(2k-1)$$

1._____

2) A line $x = k$ intersects the graph of $y = \log_5(x)$ and the graph of $y = \log_5(x+4)$. The distance between the points of intersection is 0.5. Find k.

2._____

3._____

3) An ellipse has foci at $(0,0)$ and $(14,0)$ and passes through the vertex of the parabola with equation $y = x^2 - 10x + 37$. Find the length of the major axis of the ellipse.