WMML
Meet #3
Dec. 4, 2018

Name _____

School _____

Arithmetic and Number Theory

1) How many integers between -120 and 120 are 2 more than a multiple of 7?

1._____34_____

$$-120 \leq 2 + 7n \leq 120$$
$$-17\frac{3}{7} \leq n \leq 16\frac{6}{7}$$
$$17 + 1 + 16 = 34$$

2) Lucias, Sarah, Brandon, and Scott take Number Theory together. On their first exam, Lucias got a 94, Sarah got a 91, Brandon got a 95, and Scott's score was between 81 and 87 inclusive. If the average of all four of their scores is an integer, what was Scott's score in the first exam?

2._____84_____

$94 + 91 + 95 = 280$ is divisible by 4. Scott's score must also be divisible by 4, so his score was 84.

3) A six-digit number is formed by repeating a three digit number; for example, 256256 or 678678. Find the greatest common factor of all numbers of this form.

3._____1001_____

$$abcabc = abc000 + abc = abc(1000 + 1) = abc(1001)$$

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Algebra 1

1) A parabola has vertex $(4,3)$ and its axis of symmetry parallel to the *y*-axis. If one *x*-intercept is (1,0), find the other *x*-intercept.

1._____(7,0)_____

$$4-1=3$$
$$4+3=7$$
$$(7,0)$$

2) Country A has $c\%$ of the world's population and owns $d\%$ of the world's wealth. Country B has $e\%$ of the world's population and owns $f\%$ of its wealth. Assume that the citizens of each country share their respective wealth equally. Find the ratio of the wealth of a citizen of A to the wealth of a citizen of B.

2._____ $\frac{de}{cf}$

x = population of the world, y = wealth of the world

$$\frac{dy}{cx}\div\frac{fy}{ex}=\frac{dy}{cx}\times\frac{ex}{fy}=\frac{de}{cf}$$

3) Find all values of *z* such that

3._____ $\frac{1}{2}$

$$\frac{z}{z-1}=\frac{z+1}{z}-4$$

$$z^2=(z+1)(z-1)-4(z-1)(z)$$
$$4z^2-4z+1=0$$
$$\frac{4\pm\sqrt{16-4(4)(1)}}{8}=\frac{1}{2}$$

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Geometry

$$48\left(\frac{4}{9}\right) = \frac{64}{3}$$

1) A gardener plans to build a fence to enclose a square garden plot. The area of the plot is 576 square feet, and he sets posts at the corners of the square. The posts along the sides are set 6 feet apart. How many posts will he use to fence the entire plot?

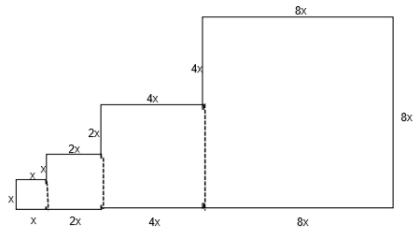
1._____16_____

$$\sqrt{576} = 24$$
$$24(4) = 96$$
$${}^{96}/_{6} = 16$$

2) The diagram below is formed by placing 4 squares together along a single line. Each square has a side length that is ½ the length of the next larger square. The outer perimeter of the figure is 115. What is the area of the whole figure, written as a simplified fraction?

$$\frac{2125}{4}$$

2._____



$$46x = 115$$
$$x = \frac{5}{2}$$
$$Area = (8x)^2 + (4x)^2 + (2x)^2 + x^2 = \frac{2125}{4}$$

3) We begin with an equilateral triangle. We divide each side into three segments of equal length, and add an equilateral triangle to each side using the middle third as a base. We then repeat this to get a third figure. Given that the perimeter of the first figure is 12, what is the perimeter of the third figure?

$$\frac{64}{3}$$

3._____

Figure 1: 3 sides of length 4

Figure 2: 12 sides of length 4/3

Figure 3: 48 sides of length 4/9

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Algebra 2

1) Find a if the remainder is a constant when $x^3 + 3x^2 + ax + 13$ is divided by $x^2 + 3x - 2$.

1._____-2_____

$$x^2 + 3x - 2 \overline{)\, x^3 + 3x^2 + ax + 13}$$

$$\begin{array}{r} x \\ -x^3 - 3x^2 + 2x \\ \hline (2+a)x + 13 \end{array}$$

$$2 + a = 0$$
$$a = -2$$

2) Simplify the following expression as much as possible:

$$\left(\frac{a^3 - 1}{a^2 - 1}\right)\left(\frac{a^2 + 2a + 1}{a^3 + 1}\right)\left(\frac{a^2 - a + 1}{a + 1}\right)$$

$$\left(\frac{(a-1)(a^2 + a + 1)}{(a+1)(a-1)}\right)\left(\frac{(a+1)(a+1)}{(a+1)(a^2 - a + 1)}\right)\left(\frac{a^2 - a + 1}{a + 1}\right)$$
$$= \frac{a^2 + a + 1}{a + 1}$$

$$\frac{a^2 + a + 1}{a + 1}$$

2._____

3) One of the roots of $x^4 + 9x^3 + 48x^2 + 78x - 136 = 0$ is $-3 + 5i$. What is the sum of all 4 roots?

3._____-9_____

$$\big(x - (-3 + 5i)\big)\big(x - (-3 - 5i)\big) = x^2 + 6x + 34$$
$$(x^4 + 9x^3 + 48x^2 + 78x - 136) \div (x^2 + 6x + 34)$$
$$= x^2 + 3x - 4 = (x + 4)(x - 1)$$

$$(-3 + 5i) + (-3 - 5i) + (-4) + 1 = -9$$

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Trigonometry and Complex Numbers

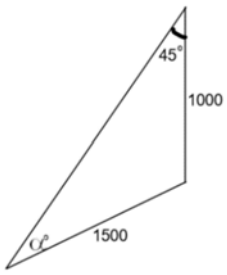
1) If $a = 3 - 5i$ and $b = 2 + i$, then what is the value of $\frac{a}{b}$ in $a + bi$ form?

$$\frac{(3-5i)}{(2+i)}\frac{(2-i)}{(2-i)} = \frac{6-3i-10i-5}{4-2i+2i+1} = \frac{1-13i}{5}$$

$$\frac{1}{5} - \frac{13}{5}i$$

1._____

2) You leave your house and walk north for 1000 feet to your friend's house. Both of you leave your friend's house and walk southwest to the library. You then turn left by $\alpha°$ to head straight back home. If your home is 1500 feet from the library, then what is $\sin(\alpha°)$?



$$\frac{\sin(45)}{1500} = \frac{\sin(\alpha)}{1000}$$

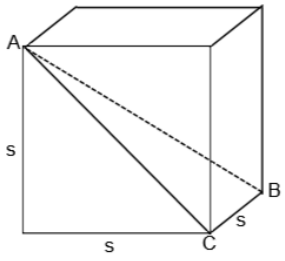
$$\frac{\frac{\sqrt{2}}{2}}{1500} = \frac{\sin(\alpha)}{1000}$$

$$\sin(\alpha) = \frac{\sqrt{2}}{3}$$

$$\frac{\sqrt{2}}{3}$$

2._____

3) *A, B,* and *C* are vertices of a cube such that $\overline{AB}$ is an interior diagonal and $\overline{AC}$ is a diagonal of a face of the cube.
Find $\sin(\angle ABC)$.



$$\overline{AC} = s\sqrt{2}$$

$$\overline{AB} = \sqrt{\left(s\sqrt{2}\right)^2 + s^2} = \sqrt{3}s^2$$

$$\sin(\angle ABC) = \frac{s\sqrt{2}}{s\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\frac{\sqrt{6}}{3}$$

3._____

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Precalculus

1) If $f(x) = \frac{x-1}{x-2}$ then what is the value of $f^{-1}(3)$?

$$3 = \frac{y-1}{y-2}$$
$$3y - 6 = y - 1$$
$$y = \frac{5}{2}$$

1.$\frac{5}{2}$

2) If $M = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$, then what is the value of M^{-1}?

$$\begin{vmatrix} 2 & 3 \\ 6 & 7 \end{vmatrix} = 2(7) - 3(6) = -4$$
$$M^{-1} = \frac{1}{-4}\begin{bmatrix} 7 & -3 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{7}{4} & \frac{3}{4} \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

2.$\begin{bmatrix} -\frac{7}{4} & \frac{3}{4} \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

3) We wish to color the integers 1,2,3,…,10 in red, green and blue, so that no two numbers a and b, with $a - b$ odd, have the same color. (Not all three colors need to be used.) In how many ways can this be done?

3.186

$$S = \{1,3,5,7,9\}, R = \{2,4,6,8,10\}$$

All 3 colors cannot be used within either of these sets. Set S can contain 1 or 2 colors.

S contains 1 color: There are $3 \cdot 2^5 = 96$ ways to color both sets.

S contains 2 colors: There are $3(2^5 - 2) = 90$ ways to color both sets (we have to exclude the cases where all odd numbers are the same color since we already counted those.

$$96 + 90 = 186$$