

WMML  
Meet #1  
Oct. 2, 2018

Name \_\_\_\_\_

School \_\_\_\_\_

Arithmetic and Number Theory

1) The 3<sup>rd</sup> term of a sequence is 5 and the 5<sup>th</sup> term is 12. Every term after the second is the sum of the two preceding terms. What is the 10<sup>th</sup> term of this sequence?

Number	1	2	3	4	5	6	7	8	9	10
Term			5	7	12	19	31	50	81	131

1. \_\_\_\_\_ 131 \_\_\_\_\_

2) What is the 2018<sup>th</sup> digit of the decimal representation for  $\frac{2}{13}$ ?

$$\frac{2}{13} = 0.\overline{153846}$$

$$2018 \bmod(6) = 2$$

The 2018<sup>th</sup> term is the second digit in the repeating decimal, so 5.

2. \_\_\_\_\_ 5 \_\_\_\_\_

3) The number 301,500 has two zeros before the first non-zero digit. How many zeros are there before the first non-zero digit in the number 2018! ?

In the prime factorization of 2018!, every 5 will yield a zero.

$$2018 \div 5 = 403 R3$$

$$2018 \div 25 = 80 R18$$

$$2018 \div 125 = 16 R18$$

$$2018 \div 625 = 3 R143$$

$$403 + 80 + 16 + 3 = 50$$

3. \_\_\_\_\_ 502 \_\_\_\_\_

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Algebra 1

1) Solve for  $x$ :

1.            $x=-4$           

$$\frac{5}{3 - \frac{x}{2}} = 1$$

$$5 = 3 - \frac{x}{2}$$

$$2 = -\frac{x}{2}$$

$$x = -4$$

2) If  $2018x - 2017 = 2016x^2 + k = 2015x - 2014$ , then find  $k$ .

2.            $k=-201$           

$$2018x - 2017 = 2015x - 2014$$

$$3x = 3$$

$$x = 1$$

$$2016(1)^2 + k = 2018(1) - 2017$$

$$2016 + k = 1$$

$$k = -201$$

3) What is the smallest possible value of  $x + y$ , where  $x$  and  $y$  are integer values that satisfy the following equation:

3.            $-101$           

$$2018^{10} \cdot 2018^{20} \cdot 2018^{30} \cdot 2018^{40} = (2018^x)^y$$

$$2018^{100} = 2018^{xy}$$

$$xy = 100$$

$$x = -100$$

$$y = -1$$

$$x + y = -101$$

Geometry

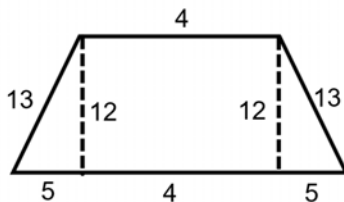
1) The lengths of the sides of a triangle are 2, 5, and 8. What is the greatest possible perimeter of a similar triangle, one of whose sides is 2018?

1. 15,135

$$\begin{aligned}2018 \div 2 &= 1009 \\2 + 5 + 8 &= 15 \\15(1009) &= 15,135\end{aligned}$$

2) Determine the area of an isosceles trapezoid with side lengths 13, 4, 14, and 13.

2. 108



$$\begin{aligned}A &= lw + 2\left(\frac{1}{2}\right)bh \\A &= 4(12) + 2\left(\frac{1}{2}\right)(5)(12) \\A &= 48 + 60 = 108\end{aligned}$$

3) Suppose a square of side length  $M$  is stretched into a rectangle by lengthening one pair of opposite sides by 20 inches and the other pair of opposite sides by 18 inches. If the area of the resulting rectangle is 440 square inches, what is the perimeter of the original square?

3. 8

$$\begin{aligned}(x + 20)(x + 18) &= 440 \\x^2 + 38x + 360 &= 440 \\x^2 + 38x - 80 &= 0 \\(x + 40)(x - 2) &= 0 \\x &= 2\end{aligned}$$

$$\text{Perimeter} = 4(2) = 8$$

Algebra 2

1) The number of free throw shots a player makes varies directly with number of shots they attempt. If the player makes three shots out of 15 attempts, how many shots should they expect to make out of 2020 attempts?

1. 404

$$\frac{3}{15} = \frac{x}{2020}$$

$$x = 404$$

2) If  $x - 3$  is a linear factor of the polynomial  $x^5 + 3x^3 - 24x^2 + 6k$ , then what is the value of  $k$ ?

2. k=-18

$$\begin{array}{r} \underline{3} \overline{) 1 \quad 0 \quad 3 \quad -24 \quad 0 \quad 6k} \\ \underline{3 \quad 9 \quad 36 \quad 36 \quad 108} \\ 1 \quad 3 \quad 12 \quad 12 \quad 36 \quad 6k+108 \end{array}$$

$$6k + 108 = 0$$

$$6k = -108$$

$$k = -18$$

3) Three whole numbers, when added together two at a time, have sums of 419, 482, and 499. What is the largest of the original three numbers?

3. z=281

$$\begin{array}{r} x + y = 419 \\ y + z = 482 \\ x + z = 499 \end{array}$$

$$\begin{array}{r} x + y = 419 \\ -(x + z = 499) \\ \hline y - z = -80 \\ y + z = 482 \\ \hline 2y = 402 \\ y = 201 \end{array}$$

$$\begin{array}{r} x + 201 = 419 \\ x = 218 \\ 201 + z = 482 \\ z = 281 \end{array}$$

Trigonometry and Complex Numbers

1) Solve for x and y:

$$(2x + y) + (2x + 3y)i = 7 + 9i$$

$$\begin{array}{r} -(2x + y = 7) \\ 2x + 3y = 9 \\ \hline 2y = 2 \\ y = 1 \end{array} \qquad \begin{array}{r} 2x + 1 = 7 \\ 2x = 6 \\ x = 3 \end{array}$$

1.           x = 3, y = 1          

2) If  $x = 5 - i$  and  $y = 2 + i$ , express  $(x - y)^3$  in  $a + bi$  form.

$$5 - i - 2 - i = 3 - 2i$$

$$(3 - 2i)(3 - 2i) = 9 - 6i - 6i - 4 = 5 - 12i$$

$$(5 - 12i)(3 - 2i) = 15 - 10i - 36i - 24 = -9 - 46i$$

2.           -9 - 46i          

3) After watching the Patriots win their football game at Gillette Stadium on Sunday, Mike drove North at 40 mph and Andrew drove Southwest at  $40\sqrt{2}$  mph. If they both arrived home in exactly 75 minutes, how far apart do they live?

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos(C) \\ c^2 &= 50^2 + (50\sqrt{2})^2 - 2(50)(50\sqrt{2}) \cos(135) \\ c^2 &= 2500 + 5000 - 5000\sqrt{2} \left(-\frac{\sqrt{2}}{2}\right) \\ c^2 &= 7500 + 5000 \\ c^2 &= 12500 \\ c &= \sqrt{12500} = 50\sqrt{5} \end{aligned}$$

3.           50√5

Precalculus

1) Consider the arithmetic sequence 2, 15, 28, 41... If 2 is the first term and 535 is the  $n^{\text{th}}$  term, what is the value of  $n$ ?

1.            $k = 42$           

$$\begin{aligned}a_k &= 13k - 11 \\535 &= 13k - 11 \\13k &= 546 \\k &= 42\end{aligned}$$

2) What is the distance between the point (2,-7) and the line  $y = \frac{1}{2}x + 2$ ? Express your answer as a radical in simplest form.

2.            $4\sqrt{5}$           

$$\begin{aligned}y + 7 &= -2(x - 2) \\y &= -2x - 3\end{aligned}$$

$$\begin{aligned}-2x - 3 &= \frac{1}{2}x + 2 \\x &= -2\end{aligned}$$

$$d = \sqrt{(2 - (-2))^2 + (-7 - 1)^2} = 4\sqrt{5}$$

3) Let  $W = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ ,  $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , and  $L = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ .

3.            $-4$           

If  $WM - ML = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then what is the value of  $a + b + c + d$ ?

$$\begin{aligned}WM &= \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 9 & 14 \end{bmatrix} \\ML &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 \\ 9 & 14 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix} &= \begin{bmatrix} -3 & -3 \\ -1 & 3 \end{bmatrix} \\ -3 + (-3) + (-1) + 3 &= -4\end{aligned}$$