WMML
Meet #4
January 9, 2018
Arithmetic and Number Theory

Name ____________________

School ____________________

1) Evaluate $16 \div \left(-\frac{2}{3}+2\right)\left(\frac{1}{4}\right)+5$

1) ____8____

$$16 \div \left(\frac{4}{3}\right)\left(\frac{1}{4}\right)+5=16\times\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)+5=3+5=8$$

2) What is the smallest composite number greater than or equal to 4 that is a factor of 291,834,015?

2) ____15____

$2+9+1+8+3+4+0+1+5=33$ so divisible by 3, but not by 9
Ends in 5 so divisible by 5
$3\times5=15$

3) If $x=3t^2+2t+1$, evaluate $\sqrt{x\left(2t^2-1\right)}$ when $t=3$. Simplify your answer completely.

3) ____$17\sqrt{2}$____

$$x=3(3)^2+2(3)+1=34$$

$$\sqrt{34\left(2(3)^2-1\right)}=\sqrt{34(17)}=\sqrt{2\times17\times17}=17\sqrt{2}$$

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Meet #4
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Algebra I

Name ______________

School ______________

1) A number is cut in half and then added to $\frac{8}{3}$. This sum is equal to $\frac{9}{4}$ less than $\frac{3}{5}$ of the number. What is the number?

1) $\frac{295}{6}$

$$60\left(\frac{1}{2}x+\frac{8}{3}\right)=60\left(\frac{3}{5}x-\frac{9}{4}\right)$$
$$30x+160=36x-135$$
$$6x=295$$
$$x=\frac{295}{6}$$

2) The force of gravity on the surface of a planet is directly proportional to the mass of the planet and is inversely proportional to the square of the radius of the planet. Planet Wimmel has twice the mass of earth and 1/3 of its radius. If a person weights 100 pounds on earth, how much does she weigh on Wimmel?

2) 1800

$$F=\frac{kM}{R^2}$$
$$\frac{2M}{\frac{1}{9}R^2}\times 100=18\times 100=1800$$

3) Jake took an exam that had 200 multiple choice questions. Each correct answer earned him 4 points and each incorrect Deducted 2 points from his score. If he answered every question on the exam and his score was 536, how many questions did he answer incorrectly?

3) 44

$$\begin{aligned} x + y &= 200 \\ 4x - 2y &= 536 \end{aligned} \qquad \begin{aligned} 2x + 2y &= 400 \\ 4x - 2y &= 536 \end{aligned}$$

$$6x=936$$
$$x=156$$
$$200-156=44$$

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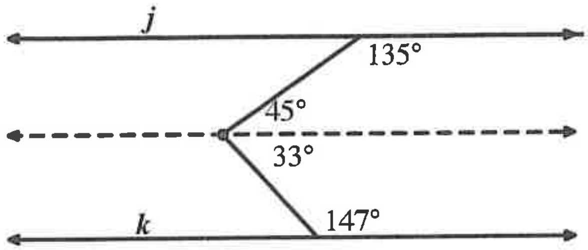
Geometry

Name ____________________

School ____________________

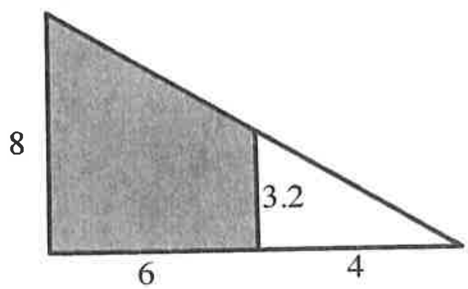
1) Determine the value or x if $j \parallel k$

1) 78°



2) Determine the area of the shaded region. The triangles are both right triangles.

2) 33.6



$$\frac{8}{10}=\frac{x}{4}$$
$$x=3.2$$
$$A=\frac{1}{2}(10)(8)-\frac{1}{2}(4)(3.2)=40-6.4=33.6$$

3) In a right rectangular prism the length, width and height are in the ratio 4:2:1. The volume of the prism is 512 cubic units. What is the surface area of the prism?

3) 448

$$(4x)(2x)(1x)=512$$
$$8x^3=512$$
$$x=4$$

$$SA=2(16)(8)+2(16)(4)+2(8)(4)=448$$

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Algebra II

Name ______________________

School ______________________

1) Factor completely: $64x^6 - 1$

$$\left(8x^3-1\right)\left(8x^3+1\right)=\left(2x-1\right)\left(4x^2+2x+1\right)\left(2x+1\right)\left(4x^2-2x+1\right)$$

1) $\left(2x-1\right)\left(4x^2+2x+1\right)\left(2x+1\right)\left(4x^2-2x+1\right)$

2) Simplify $\left[\left(2x^3+x^2-25x+12\right)\div\left(x-3\right)\right]\div\left(2x-1\right)$

2) $x+4$

3	2	1	−25	12
		6	21	−12
	2	7	−4	0

$$\frac{2x^2+7x-4}{2x-1}=\frac{\left(2x-1\right)\left(x+4\right)}{2x-1}=x+4$$

3) If $f(x)=\dfrac{\sqrt{x}}{2+\sqrt{x}}$ and $g(x)=\left(\dfrac{2x}{1-x}\right)^2$, compute $g\left(f\left(x\right)\right)$.
Simplify your answer completely.

3) x

$$g\left(f\left(x\right)\right)=\left(\frac{2\left(\frac{\sqrt{x}}{2+\sqrt{x}}\right)}{1-\frac{\sqrt{x}}{2+\sqrt{x}}}\right)^2=\left(\frac{\frac{2\sqrt{x}}{2+\sqrt{x}}}{\frac{2+\sqrt{x}-\sqrt{x}}{2+\sqrt{x}}}\right)^2=\left(\frac{2\sqrt{x}}{2}\right)^2=\left(\sqrt{x}\right)^2=x$$

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Trigonometry and Complex Numbers

Name ______________________

School ______________________

1) If $\tan x + \cot x = \frac{144}{25}$, determine the value of $\frac{1}{\tan x} + \frac{1}{\cot x}$.

1) $\frac{144}{25}$

$$\frac{1}{\tan x} + \frac{1}{\cot x} = \frac{\cot x + \tan x}{\tan x \cot x} = \frac{\frac{144}{25}}{1} = \frac{144}{25}$$

2) Solve for x if $0 \le x < 2pi$: $\sin^2 x + \cos x = -1$

2) *pi*

$$1 - \cos^2 x + \cos x = -1$$
$$\cos^2 x - \cos x - 2 = 0$$
$$(\cos x - 2)(\cos x + 1) = 0$$
$$\cos x = 2, \quad \cos x = -1$$
$$\varnothing, \qquad x = pi$$

3) At what values of x $(-12 < x < 12)$, does the graph of $y = 2\sec\left[\frac{pi}{6}(x-1)\right] + 3$ have a vertical asymptote?

3) *y*= -8, -2, 4, and 10

The graph of $y = \sec x$ has vertical asymptotes at $y = \frac{pi}{2} + k(pi)$. This graph is stretched horizontally by a factor of $\frac{6}{pi}$, so the asymptotes will occur 6 units apart. The asymptote originally located at $y = -\frac{pi}{2}$ will be "stretched" to $y = -3$ and then shifted to the right one unit to $y = -2$. Since the asymptotes occur every 6 units, in the interval noted, they will occur at *y*= -8, -2, 4, and 10.

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Meet #4
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Precalculus

Name ______________________

School ______________________

1) For $A=\begin{bmatrix}3&1&0\\6&4&0\\2&3&1\end{bmatrix}$ and $B=\begin{bmatrix}2&1&0\\3&3&9\\6&4&6\end{bmatrix}$, determine $B-2A$

1) $\begin{bmatrix}-4&-1&0\\-9&-5&9\\2&-2&-12\end{bmatrix}$

$$\begin{bmatrix}2&1&0\\3&3&9\\6&4&6\end{bmatrix}-2\begin{bmatrix}3&1&0\\6&4&0\\2&3&1\end{bmatrix}=\begin{bmatrix}2&1&0\\3&3&9\\6&4&6\end{bmatrix}-\begin{bmatrix}6&2&0\\12&8&0\\4&6&12\end{bmatrix}=\begin{bmatrix}-4&-1&0\\-9&-5&9\\2&-2&-12\end{bmatrix}$$

2) If $\begin{bmatrix}3&0&1\\-2&3&0\end{bmatrix}\times\begin{bmatrix}a&c\\-b&a\\c&b\end{bmatrix}=\begin{bmatrix}6&7\\4&-3\end{bmatrix}$, determine the value of $a+b+c$.

2) 2

$3a+c=6$
$3c+b=7$
$-2a-3b=4$
$-2c+3a=-3$

$c=6-3a$
$-2(6-3a)+3a=-3$
$-12+6a+3a=-3$
$9a=9$
$a=1$

$c=6-3(1)=3$
$3(3)+b=7$
$b=-2$
$a+b+c=1-2+3=2$

3) Solve over the set of real numbers.

3) 6

$$\det\begin{bmatrix}2x&3x-2&0\\3&x&1\\1-x&2&x\end{bmatrix}=40$$

$$2x\begin{vmatrix}x&1\\2&x\end{vmatrix}-(3x-2)\begin{vmatrix}3&1\\1-x&x\end{vmatrix}=40$$

$2x(x^2-2)-(3x-2)(3x-(1-x))=40$

$2x^3-4x-(3x-2)(4x-1)=40$

$2x^3-4x-12x^2+11x-2=40$

$2x^3-12x^2+7x-42=0$

$2x^2(x-6)+7(x-6)=0$

$(2x^2+7)(x-6)=0$

The only real solution to this equation is $x=6$

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Team Round

Name ____________________

School ____________________

1) Several people are standing in a line. Starting from one end, Angie is the 4th person in line and from the other end she is the 13th person in line. How many people are in the line?

$4 + 13 - 1 = 16$

1) 16

2) Solve for y in terms of x.

$$2x+2y=3(4-6yx)-4(-x)-12y$$
$$2x+2y=12-18yx+4x-12y$$
$$-2x-12=-14y-18xy$$
$$-2x-12=y(-14-18x)$$
$$y=\frac{-2x-12}{-18x-14} \text{ or } \frac{x+6}{9x+7}$$

2) $\frac{-2x-12}{-18x-14}$ or $\frac{x+6}{9x+7}$

3) In $\triangle ABC$, $AB=5$, $BC=12$, and $\overline{AB}\perp\overline{BC}$. Determine the distance from B to $\overline{AC}$.

Area of $\triangle ABC=\frac{1}{2}(5)(12)=30$

$$30=\frac{1}{2}(x)(13)$$
$$x=\frac{60}{13}$$

3) $\frac{60}{13}$

4) If $9x^3+5x-k$ is divide by $3x-2$, the remainder is -2. Determine the value of k.

$$\begin{array}{c|cccc} \frac{2}{3} & 9 & 0 & 5 & -k \\ & & 6 & 4 & 6 \\ \hline & 9 & 6 & 9 & -k+6=-2 \end{array}$$

$k=8$

4) 8

5) Determine the exact value of $\sin(-15^\circ)$

$$\sin(-15^\circ)=-\sin(15^\circ)=-\sin(45^\circ-30^\circ)=-(\sin 45^\circ\cos 30^\circ-\sin 30^\circ\cos 45^\circ)=$$
$$-\left(\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\right)=-\frac{\sqrt{6}-\sqrt{2}}{4} \text{ or } \frac{\sqrt{2}-\sqrt{6}}{4}$$

5) $-\frac{\sqrt{6}-\sqrt{2}}{4}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

6) Determine the x-coordinate of the focus of the conic section given by $y^2+4y-6x+22=0$.

Conic is a horizontal parabola opening to the right.

$$y^2+4y-6x+22=0$$
$$y^2+4y+4-6x+22-4=0$$
$$(y+2)^2-6x+18=0$$
$$(y+2)^2=6x-18$$
$$(y+2)^2=6(x-3)$$

Vertex of the parabola is at (3, -2)

Distance from vertex to focus is $\frac{1}{4}(6)=\frac{3}{2}$

$$\frac{3}{2}+3=\frac{9}{2}$$

6) $\frac{9}{2}$